Math 445 – David Dumas – Spring 2019

Homework 1

Due Monday, January 28 in class (1:00pm)

Notes and instructions:

- Problems from the textbook (Munkres, 2ed) are specified in the format "Section.Exercise". For example, "13.8" means exercise 8 from section 13, which is the last problem on page 83.
- When writing your solutions, clearly label each one with the problem number. Use the same conventions as in the homework sheet (i.e. 13.8, 14.2, etc., for book problems, P1, P2, etc. for non-book problems).
- Please write the solutions in the same order as the problem sheet. The suggested way to achieve this while working on the problems in an arbitrary order is to work out the solutions on scratch paper, and then to rewrite them in final form and in the correct order when everything is done.
- To receive full credit, a solution must be clear, concise, correct, and written in full sentences.
- Answers to "yes or no" questions must always be accompanied by a proof.

(—) From the textbook: 13.1, 13.4 part (a), 13.8, 16.6

(P1) Let X be a set, and let $\mathscr{T}_\infty = \{U \mid X - U \text{ is infinite or empty or all of } X\}$.
Is $\mathscr{T}_\infty$ a topology on X?

(P2) Show that any open subset of $\mathbb{R}$ (with the standard topology) is a *countable* union of open intervals. You may make use of the following fact about the reals which is typically proved in an analysis course: If $a, b \in \mathbb{R}$ and $a < b$, then there exists a rational number $q \in \mathbb{Q}$ with $a < q < b$.

(P3) Suppose that X and Y are discrete topological spaces. Is $X \times Y$ discrete?