

Midterm Exam

Instructions:

- Complete **three** of these problems. (If you work on more than three problems—which is not recommended—then only your three highest problem scores will count toward your grade.)
- Clearly label each of your solutions with the problem number.
- Each question implicitly ends with “and give a proof of your answer”.

(1) Is \mathbb{R} connected in the lower limit topology?

(2) Consider these topologies on $\mathbb{Z} \times \mathbb{Z}$:

- The cofinite topology (on the set $\mathbb{Z} \times \mathbb{Z}$)
- The product of the cofinite topology on \mathbb{Z} and the cofinite topology on \mathbb{Z}

Are these topologies the same? Is one finer than the other?

(3) Let \mathcal{T} denote the topology on \mathbb{R} generated by the basis consisting of intervals $[a, b)$ with **rational** endpoints, i.e. where $a, b \in \mathbb{Q}$. Determine the closure of $(\sqrt{2}, \sqrt{3})$ in this topology.*

(4) Is $[0, 1]^{\mathbb{N}}$ a closed set in $\mathbb{R}^{\mathbb{N}}$ with respect to the uniform topology?

(Partial credit will be given for just writing the definition of the uniform metric on $\mathbb{R}^{\mathbb{N}}$.)

* There was a typographical error in this problem: The original version had the endpoints of the interval switched.