Math 550 - David Dumas - Spring 2019

Project Suggestions

Most of these are broad topic areas in which you would need to determine a more specific topic of manageable scope for your project. Also, proposals for projects on topics not listed here are certainly welcome.

- 1. The universal principal *G*-bundle $EG \rightarrow BG$; identification of isomorphism classes of principal bundles over *B* with homotopy classes of maps $B \rightarrow BG$.
- 2. The only spheres that have Lie group structures are S^1 and S^3 .
- 3. Compact 3-manifolds that are fiber bundles over S^1 in infinitely many ways.
- 4. Symmetric spaces (as examples of G/H with G-invariant Riemannian metric)
- 5. The space of connections on a given principal *G*-bundle (for *G* compact) as an "infinitedimensional symplectic manifold" with the Atiyah-Bott form.
- 6. Holomorphic structures on vector bundles over complex manifolds; equivalent descriptions as $\bar{\partial}$ operator on sections, or by holomorphic local trivialization.
- 7. Symmetric spaces (as examples of G/H with G-invariant Riemannian metric).
- 8. Classification of compact simple Lie groups.
- 9. De Rham cohomology of a compact Lie group.
- 10. On a 3-manifold, every homotopy class of rank-2 distributions contains an integrable one (Thurston).
- 11. Existence of sphere eversions.
- 12. Seifert fiber spaces.

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