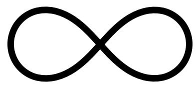
Math 547: Algebraic Topology I – David Dumas – Fall 2023

Final Exam

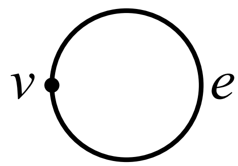
(P1) Let $A \simeq S^1 \vee S^1$ be the figure-8 space shown below. Regard it as a subset of $\mathbb{R}^3$ by considering this drawing to be in the xy plane.



(a) [5 points] Compute the homology groups $H_i(A)$.

(b) [5 points] Compute the homology groups $H_i(\mathbb{R}^3/A)$. Note that $\mathbb{R}^3/A$ means the quotient space where A is collapsed to a point.

(P2) Let X be a CW complex with one cell of each dimension $0,1,2,3$, with respective names v, e, f, b (for "vertex", "edge", "face", "ball"). Thus the 1-skeleton X^1 is as shown below, and in particular is homeomorphic to S^1.



The other cells are attached as follows:

- The 2-cell f is attached by a map $\partial D^2 \simeq S^1 \longrightarrow S^1 \simeq X^1$ of degree 5.
- The 3-cell b is attached by gluing the entire boundary $\partial D^3 \simeq S^2$ to the point v.

(a) [5 points] Compute the homology groups $H_i(X;\mathbb{Z}/3\mathbb{Z})$.

(b) [5 points] Compute the homology groups $H_i(X;\mathbb{Z}/5\mathbb{Z})$.

(P3) [10 points] Let X be a simply connected space that is locally homeomorphic to $\mathbb{R}^4$. Let $x_0 \in X$. Show that $X - \{x_0\}$ is simply connected.

(P4) Let's say that a space X *retracts to a circle* if there exists a subset $A \subset X$ that is homeomorphic to S^1 and a retraction $r : X \to A$.

(a) [8 points] Show that no covering space of $\mathbb{RP}^2 \times \mathbb{RP}^2$ retracts to a circle.

(b) [2 points] Find a covering space of $\mathbb{RP}^2 \vee \mathbb{RP}^2$ that retracts to a circle.