Math 547: Algebraic Topology I – David Dumas – Fall 2023

Homework 1

Due Wednesday September 6 at 11:59pm (note unusual day)

Instructions:

- Write solutions with complete sentences that explain your answer. Words are needed; formulas and drawings alone are not enough.
- Write as if the audience is another student in the class.
- Do not include this document as part of what you submit.
- Problems from Hatcher are indicated by numbers in the format *[chapter].[section].[problem]*. Chapter 0 doesn't have sections, though.
- Label your solution with the same number listed here, i.e. the full textbook problem number or something like (P2) for non-book problems.
- Submit your solutions to Gradescope. Please use Gradescope's system that lets you indicate the correspondence between problems and pages of your submission.

Note that I will include these full instructions on the first two assignments, after which they will become the "standing instructions" that apply all the time.

Problems: ($*$ means expected to be more challenging)

– 0.3

– 0.4

– 0.5

– 0.6

– 0.16*

Hint: First show Id_{S^∞} is homotopic to the map $f(x_1,x_2,x_3,\ldots) = (0,x_1,x_2,x_3,\ldots)$, then show f is homotopic to the constant map with value $(1,0,0,0,\ldots)$. That is, the suggested way to approach this problem will *not* give a deformation retraction to a point.

(P1) Give an example (with proof) of a compact subspace of $\mathbb{R}$ that is not homeomorphic to any CW complex. (Hint: You can use Proposition A.1 from the appendix in Hatcher.)