Math 547: Algebraic Topology I – David Dumas – Fall 2023

Homework 6

Due Monday October 9 at 11:59pm

Instructions: *Same as in Homework 2.*

Problems:

- 1.3.12 (Note this problem talks about the *normal* subgroup generated by a certain list of elements, which is much larger than the subgroup generated by those elements. Also, you can use Proposition 1A.2 which we'll talk about in Lecture 19.)

- 1.3.15

(P1) Suppose $G \curvearrowright X$ is an even action and that $H \subset G$ is a subgroup. It is immediate that the induced action $H \curvearrowright X$ is even as well. Show that the map $X/H \to X/G$ that takes $H \cdot x$ to $G \cdot x$ is a covering map of degree equal to the index of H in G. (This generalizes the fact that $X \to X/G$ is a covering map.)

(P2) Describe all isomorphism classes of 4-sheeted covers of $S^1 \times \mathbb{RP}^2$. Note we don't assume the cover is connected here. Also you can use that $\mathbb{RP}^2$ is the quotient of S^2 by the even action of $\mathbb{Z}/2$ generated by $v \mapsto -v$.

(P3) As discussed in Lecture 18, covering spaces of $S^1 \vee S^1$ can be described by drawing graphs with edges oriented and labeled by two symbols (e.g. a and b), subject to the condition that at each vertex there is one incoming and one outgoing edge of each label. For the covers of $S^1 \vee S^1$ described in that manner below, determine whether the covering is regular or not. If it is a regular cover, describe the automorphism group as an abstract group (e.g. cyclic, dihedral, something else) and draw a diagrams showing the action of a set of automorphisms that generate the group.