Homework 7

Due Monday October 30 at 11:59pm

Instructions: *Same as in Homework 2.*

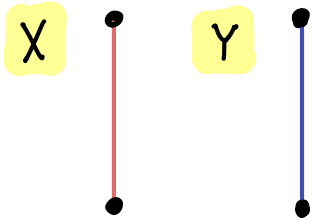**Problems:**

– 1.2.1

– 1.2.3

– 1.2.7

– 1.2.10

– 1.2.21†

† For this problem, we need to introduce an operation on spaces that was described in Chapter 0 of Hatcher's book but never discussed in lecture.

If X and Y are topological spaces, the *join* $X * Y$ is the quotient of $X \times Y \times I$ by the equivalence relation generated by:
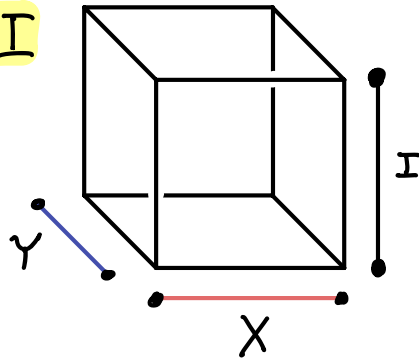
$$(x,y,0) \sim (x,y',0) \quad \text{for all } x \in X \text{ and } y,y' \in Y$$
$$(x,y,1) \sim (x',y,1) \quad \text{for all } x,x' \in X \text{ and } y \in Y$$

There's a discussion of this and a picture of the case $X = Y = I$ on page 9 of Hatcher's book. I added my own sketch of the construction on the next page. And here are two intuitive descriptions that might help to clarify the construction:
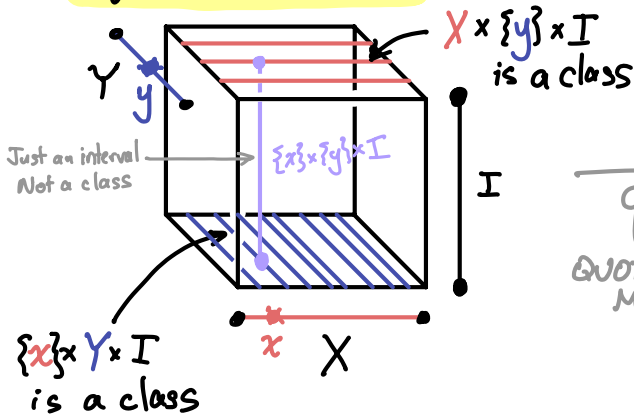
- $X \times Y \times I$ is an interval of copies of $X \times Y$ parameterized by $t \in I$, which you can think of as a stack with $t = 0$ at the bottom and $t = 1$ at the top. In $X * Y$ you modify the ends of that stack, collapsing the bottom $X \times Y$ to X and the top $X \times Y$ to Y.

- $X * Y$ is a space that contains $X \sqcup Y$, but where there is also a line segment joining any point $x \in X$ to any point $y \in Y$; that segment is the image of $\{x\} \times \{y\} \times I$ under the quotient map $X \times Y \times I \to X * Y$.

X

Y

X × Y × I

I

Y

X

Equivalence Classes

X × {y} × I
is a class

Just an interval
Not a class

{x} × {y} × I

Y
y

I

{x} × Y × I
is a class

x

X

X ∗ Y (the Quotient)

q
QUOTIENT
MAP

image of that
interval in X∗Y